Shear Design Example

Design the shear reinforcement for the following beam:

\[ f'_c = 4 \text{ kip/in}^2 \quad d' = 2.5 \text{ in} \]
\[ f_y = 60 \text{ kip/in}^2 \quad d = 22.5 \text{ in} \]
\[ \ell = 24 \text{ ft} \quad d_t = 24 \text{ in} \]
\[ b = 14 \text{ in} \quad A'_s = 2.40 \text{ in}^2 (4 – \#7) \]
\[ h = 27 \text{ in} \quad A_s = 8 \text{ in}^2 (8 – \#9) \]
\[ SDL = 2.5 \text{ kip/ft} \quad w_L = 3.5 \text{ kip/ft} \]

1. Find the shear envelope.

\[ SW = \text{ Self Weight} = bh\gamma_c \quad (1) \]
\[ = 14 \text{ in}(27 \text{ in})(150 \text{ lb/ft}^3)(1 \text{ foot}/12 \text{ in})^2(1 \text{ kip}/1000 \text{ lb}) = 0.39 \text{ kip/ft} \quad (2) \]
\[ w_D = SW + SDL = 0.39 \text{ kip/ft} + 2.5 \text{ kip/ft} = 2.89 \text{ kip/ft} \quad (3) \]

By inspection, the dead load is less than eight times the live load, so \( w = 1.4DL \) need not be checked.

\[ w_u = 1.2w_D + 1.6w_L \quad (4) \]
\[ = 1.2(2.89 \text{ kip/ft}) + 1.6(3.5 \text{ kip/ft}) = 9.07 \text{ kip/ft} \quad (5) \]

Now calculate the maximum factored shear at the support (\( V_{u,\text{sup}} \)) and midspan (\( V_{u,\text{mid}} \)).

\[ V_{u,\text{sup}} = w_u\ell/2 = (9.07 \text{ kip/ft})(24 \text{ ft})/2 = 108.9 \text{ kip} \quad (6) \]
\[ V_{u,\text{mid}} = 1.6(w_L)\ell/8 = 1.6(3.5 \text{ kip/ft})(24 \text{ ft})/8 = 16.8 \text{ kip} \quad (7) \]

The formula for the shear as a function of the distance, \( x \), from the support is given by:

\[ V_u(x) = V_{u,\text{sup}} - \frac{(V_{u,\text{sup}} - V_{u,\text{mid}})x}{\ell/2} \quad (8) \]
\[ = 108.9 \text{ kip} - \frac{(108.9 \text{ kip} - 16.8 \text{ kip})x}{(24 \text{ ft})/2} \quad (9) \]
\[ = 108.9 \text{ kip} - (7.68 \text{ kip/ft})x \quad (10) \]

At the critical section, distance \( d \) from the support:

\[ V_u(d) = V_u(22.5 \text{ in}) = 108.9 \text{ kip} - (7.68 \text{ kip/ft})(22.5 \text{ in})(1 \text{ foot}/12 \text{ in}) \]
\[ = 94.5 \text{ kip} = V_{u,\text{max}} \quad (12) \]

The resulting shear envelope is shown in Figure 1.

2. Check maximum shear reinforcing steel requirement.

\[ V_c = 2\sqrt{f'_c bd} \quad (13) \]
\[ = 2\sqrt{4,000 \text{ lb/in}^2(14 \text{ in})(22.5 \text{ in})(1 \text{ kip}/1000 \text{ lb})} = 39.8 \text{ kip} \quad (14) \]
\[ \phi(V_c + 8\sqrt{f'_c bd}) \]
\[ = 0.75(39.8 \text{ kip} + 8\sqrt{4,000 \text{ lb/in}^2(14 \text{ in})(22.5 \text{ in})(1 \text{ kip}/1000 \text{ lb})}) \]
\[ = 149 \text{ kip} > V_{u,\text{max}} = 94.5 \text{ kip} \quad \text{O.K.} \quad (16) \]

The section is of adequate size for the shear reinforcing.
3. Pick stirrup size and set spacing. To begin, we determine the distance from the support to the end of Zone C.

\[
\phi V_c = 0.75(39.8 \text{kip}) = 29.9 \text{kip} \tag{17}
\]

\[
V_u(x) = 108.9 \text{kip} - (7.68 \text{kip/ft})x = \phi V_c = 29.9 \text{kip} \tag{18}
\]

\[
\Rightarrow x = \frac{(108.9 \text{kip} - 29.9 \text{kip})}{(7.68 \text{kip/ft})} = 10.3 \text{ ft} \tag{19}
\]

Zone C ends 10.3 feet from the support, which is shown graphically by the dotted line in Figure 1. Try a #3 stirrup \((A_b = 0.11 \text{ in}^2 \Rightarrow A_v = 0.22 \text{ in}^2)\). Use \(V_{u,\text{max}}\) to calculate the stirrup spacing, \(s\).

\[
s = \frac{A_v f_y d}{V_{u,\text{max}} - V_c} \tag{20}
\]

\[
= \frac{0.22 \text{ in}^2(60 \text{kip/in}^2)(22.5 \text{ in})}{94.5 \text{ kip/0.75} - 39.8 \text{ kip}} \tag{21}
\]

\[
= 3.45 \text{ in} \tag{22}
\]

Spacing is inadequate: it is best to use \(s \geq 4 \text{ in}\). Try a #4 stirrup \((A_b = 0.20 \text{ in}^2 \Rightarrow A_v = 0.40 \text{ in}^2)\):

\[
s = \frac{0.40 \text{ in}^2(60 \text{kip/in}^2)(22.5 \text{ in})}{94.5 \text{ kip/0.75} - 39.8 \text{ kip}} \tag{23}
\]

\[
= 6.26 \text{ in} \tag{24}
\]

Use \(s = 6 \text{ in}\) as the stirrup spacing. The shear capacity of the stirrups, \(V_s\) is then calculated as:

\[
V_s = \frac{A_v f_y d}{s} = \frac{0.40 \text{ in}^2(60 \text{kip/in}^2)(22.5 \text{ in})}{6 \text{ in}} = 90 \text{ kip} \tag{25}
\]

This value must be compared to \(4\sqrt{f'_c bd}\) to check maximum spacing:

\[
4\sqrt{f'_c bd} = 4\sqrt{4,000 \text{ lb/in}^2(14 \text{ in})(22.5 \text{ in})1 \text{ kip}/1000 \text{ lb} = 79.7 \text{ kip} \tag{26}
\]

Figure 1: The shear envelope.
Thus, because $V_s > 4\sqrt{f_c'bd}$, the spacing must satisfy

$$s \leq \min\left(\frac{d}{4}, 12\text{ in}\right) = \min\left(\frac{(22.5\text{ in})}{4}, 12\text{ in}\right) = \left(\frac{22.5\text{ in}}{4}\right) = 5.63\text{ in} \quad (27)$$

Use an initial spacing of 5 inches to meet the maximum spacing of 5.6 inches. We should change spacing at most two or three times. If we increase spacing to $s = 8\text{ in}$:

$$V_s = \frac{0.40\text{ in}^2(60\text{ kip/in}^2)(22.5\text{ in})}{8\text{ in}} = 67.5\text{ kip} \quad (28)$$

Because the shear capacity, $V_s$, at $s = 8\text{ in}$ is less than $4\sqrt{f_c'bd} = 79.7\text{ kip}$, the maximum spacing is governed by:

$$s \leq \min\left(\frac{d}{2}, 24\text{ in}\right) = \min\left(\frac{(22.5\text{ in})}{2}, 24\text{ in}\right) = \left(\frac{22.5\text{ in}}{2}\right) = 11.25\text{ in} \quad (29)$$

Thus $s = 8\text{ in}$ can be used as soon as $V_u(x)$ drops low enough:

$$\phi(V_c + V_s) = 0.75(39.8\text{ kip} + 67.5\text{ kip}) = 80.5\text{ kip} \leq V_u(x) \quad (30)$$

Solving $V_u(x) = 80.5\text{ kip}$ for $x$ gives (see Equation 10):

$$108.9\text{ kip} - (7.68\text{ kip/ft})x = 80.5\text{ kip} \quad (31)$$

$$\Rightarrow x = \frac{108.9\text{ kip} - 80.5\text{ kip}}{7.68\text{ kip/ft}} = 3.70\text{ ft} \quad (32)$$

We will also consider the maximum spacing, $s = 11\text{ in}$:

$$V_s = \frac{0.40\text{ in}^2(60\text{ kip/in}^2)(22.5\text{ in})}{11\text{ in}} = 49.1\text{ kip} \quad (33)$$

$$\phi(V_c + V_s) = 0.75(39.8\text{ kip} + 49.1\text{ kip}) = 66.7\text{ kip} \quad (34)$$

Solving as before, this spacing may begin 5.5 feet from the support.

Moving on to Zone B, the maximum spacing is $s = d/2$, or $s = 11\text{ in}$ as shown above. We also need to check the minimum steel area from \{ACI 11.5.5.3\}:

$$A_v \geq \frac{0.75\sqrt{f_c'b_w s}}{f_y} = \frac{0.75\sqrt{4,000\text{ lb/in}^2}(14\text{ in})(11\text{ in})}{60,000\text{ lb/in}^2} \quad (35)$$

$$\geq 0.12\text{ in}^2 \quad (36)$$

$$A_v \geq \frac{50b_w s}{f_y} = \frac{50\text{ psi}(14\text{ in})(11\text{ in})}{60,000\text{ lb/in}^2} \quad (37)$$

$$\geq 0.13\text{ in}^2 \quad (38)$$

The larger area ($A_v \geq 0.13\text{ in}^2$) controls. A #3 would work ($A_v = 2A_b = 2(0.11\text{ in}^2) = 0.22\text{ in}^2$), but we do not want to change bar size. Use a #4 at spacing $s = 11\text{ in}$.

Lastly, determine where Zone A starts:

$$\frac{\phi V_c}{2} = \frac{0.75(39.8\text{ kip})}{2} = 14.9\text{ kip} < V_{u,\text{mid}} = 16.8\text{ kip} \quad (39)$$

Thus, stirrups are required everywhere since there is no point on the beam where the concrete alone is twice as strong as it needs to be. That is, there is no Zone A for this problem.
4. **Sketch.**

The first stirrup will be placed a distance \( s/2 \) from the support. The initial spacing is 5 inches so that the first stirrup would be at 2.5 inches. But, we want to detail everything in whole inches so use an initial space of 2 inches. Nine additional stirrups at 5 inch spacing takes us to 47 inches past the support or 3.9 ft—a bit past the 3.70 feet calculated above in Equation 32. Four more stirrups at 8 inch spacing takes us to 71 inches (5.9 feet) and 6 more stirrups at 11 inches on center will leave a 10 inch distance to the reinforcing bar at the midspan.

**Figure 2:** Stirrup design layout.