1. Is the center of mass of the dumbbell in FIGURE Q12.1 at point a, b, or c? Explain.

FIGURE Q12.1

2. If the angular velocity \( \omega \) is held constant, by what factor must \( R \) change to double the rotational kinetic energy of the dumbbell in FIGURE Q12.2?

FIGURE Q12.2

3. FIGURE Q12.3 shows three rotating disks, all of equal mass. Rank in order, from largest to smallest, their rotational kinetic energies \( K_1 \) to \( K_3 \).

FIGURE Q12.3

4. Must an object be rotating to have a moment of inertia? Explain.

5. The moment of inertia of a uniform rod about an axis through its center is \( \frac{1}{12} mL^2 \). The moment of inertia about an axis at one end is \( \frac{1}{3} mL^2 \). Explain why the moment of inertia is larger about the end than about the center.

6. You have two steel spheres. Sphere 2 has twice the radius of sphere 1. By what factor does the moment of inertia \( I_2 \) of sphere 2 exceed the moment of inertia \( I_1 \) of sphere 1?

7. The professor hands you two spheres. They have the same mass, the same radius, and the same exterior surface. The professor claims that one is a solid sphere and the other is hollow. Can you determine which is which without cutting them open? If so, how? If not, why not?

8. Six forces are applied to the door in FIGURE Q12.8. Rank in order, from largest to smallest, the six torques \( \tau_1 \) to \( \tau_6 \) about the hinge. Explain.

FIGURE Q12.8

9. The dumbbells in FIGURE Q12.9 are all the same size, and the forces all have the same magnitude. Rank in order, from largest to smallest, the torques \( \tau_a \), \( \tau_b \), and \( \tau_c \). Explain.

FIGURE Q12.9

10. A student gives a quick push to a ball at the end of a massless, rigid rod, as shown in FIGURE Q12.10, causing the ball to rotate clockwise in a horizontal circle. The rod’s pivot is frictionless.
   a. As the student is pushing, is the torque about the pivot positive, negative, or zero?
   b. After the push has ended, does the ball’s angular velocity (i) steadily increase; (ii) increase for awhile, then hold steady; (iii) hold steady; (iv) decrease for awhile, then hold steady; or (v) steadily decrease? Explain.
   c. Right after the push has ended, is the torque positive, negative, or zero?

FIGURE Q12.10

11. Rank in order, from largest to smallest, the angular accelerations \( \alpha_1 \) to \( \alpha_4 \) in FIGURE Q12.11. Explain.

FIGURE Q12.11
12. The solid cylinder and cylindrical shell in Figure Q12.12 have the same mass, same radius, and turn on frictionless, horizontal axles. (The cylindrical shell has lightweight spokes connecting the shell to the axle.) A rope is wrapped around each cylinder and tied to a block. The blocks have the same mass and are held the same height above the ground. Both blocks are released simultaneously. Which hits the ground first? Or is it a tie? Explain.

13. A diver in the pike position (legs straight, hands on ankles) usually makes only one or one-and-a-half rotations. To make two or three rotations, the diver goes into a tuck position (knees bent, body curled up tight). Why?

14. Is the angular momentum of disk a in Figure Q12.14 larger than, smaller than, or equal to the angular momentum of disk b? Explain.
15. The four masses shown in FIGURE E12.14 are connected by massless, rigid rods.
   a. Find the coordinates of the center of mass.
   b. Find the moment of inertia about a diagonal axis that passes through masses B and D.
16. The three masses shown in FIGURE E12.16 are connected by massless, rigid rods.
   a. Find the coordinates of the center of mass.
   b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.
   c. Find the moment of inertia about an axis that passes through masses B and C.
17. A 25 kg solid door is 220 cm tall, 91 cm wide. What is the door’s moment of inertia for (a) rotation on its hinges and (b) rotation about a vertical axis inside the door, 15 cm from one edge?
18. A 12-cm-diameter CD has a mass of 21 g. What is the CD’s moment of inertia for rotation about a perpendicular axis (a) through its center and (b) through the edge of the disk?

Section 12.5 Torque

19. In FIGURE E12.19, what is the net torque about the axle?

20. In FIGURE E12.20, what is the net torque about the center of mass?

21. The tune-up specifications of a car call for the spark plugs to be tightened to a torque of 38 N\,m. You plan to tighten the plugs by pulling on the end of a 25-cm-long wrench. Because of the cramped space under the hood, you’ll need to pull at an angle of 120° with respect to the wrench shaft. With what force must you pull?

22. The 20-cm-diameter disk in FIGURE E12.22 can rotate on an axle through its center. What is the net torque about the axle?

23. A 4.0-m-long, 500 kg steel beam extends horizontally from the point where it has been bolted to the framework of a new building under construction. A 70 kg construction worker stands at the far end of the beam. What is the magnitude of the torque about the point where the beam is bolted into place?

24. An athlete at the gym holds a 3.0 kg steel ball in his hand. His arm is 70 cm long and has a mass of 4.0 kg. What is the magnitude of the torque about his shoulder if he holds his arm
   a. Straight out to his side, parallel to the floor?
   b. Straight, but 45° below horizontal?

Section 12.6 Rotational Dynamics

Section 12.7 Rotation About a Fixed Axis

25. An object’s moment of inertia is 2.0 kg\,m². Its angular velocity is increasing at the rate of 4.0 rad/s per second. What is the torque on the object?

26. An object whose moment of inertia is 4.0 kg\,m² experiences the torque shown in FIGURE E12.26. What is the object’s angular velocity at \( t = 3.0 \) s? Assume it starts from rest.

27. A 1.0 kg ball and a 2.0 kg ball are connected by a 1.0-m-long rigid, massless rod. The rod is rotating cw about its center of mass at 20 rpm. What torque will bring the balls to a halt in 5.0 s?

28. A 200 g, 20-cm-diameter plastic disk is spun on an axle through its center by an electric motor. What torque must the motor supply to take the disk from 0 to 1800 rpm in 4.0 s?

29. Starting from rest, a 12-cm-diameter compact disk takes 3.0 s to reach its operating angular velocity of 2000 rpm. Assume that the angular acceleration is constant. The disk’s moment of inertia is \( 2.5 \times 10^{-5} \) kg\,m².
   a. How much torque is applied to the disk?
   b. How many revolutions does it make before reaching full speed?

30. The 200 g model rocket shown in FIGURE E12.30 generates 4.0 N of thrust. It spins in a horizontal circle at the end of a 100 g rigid rod. What is its angular acceleration?

Section 12.8 Static Equilibrium

31. How much torque must the pin exert to keep the rod in FIGURE E12.31 from rotating?

32. Is the object in FIGURE E12.32 in equilibrium? Explain.
33. The two objects in FIGURE EX12.33 are balanced on the pivot. What is distance d?

![Figure EX12.33](image)

34. \[ A \text{ 5.0 kg cat and a 2.0 kg bowl of tuna fish are at opposite ends of a 4.0-m-long seesaw. How far to the left of the pivot must a 4.0 kg cat stand to keep the seesaw balanced?} \]

![Figure EX12.34](image)

Section 12.9 Rolling Motion

35. A car tire is 60 cm in diameter. The car is traveling at a speed of 20 m/s.
   a. What is the tire’s rotation frequency, in rpm?
   b. What is the speed of a point at the top edge of the tire?
   c. What is the speed of a point at the bottom edge of the tire?

36. A 500 g, 8.0-cm-diameter can rolls across the floor at 1.0 m/s. What is the can’s kinetic energy?

37. An 8.0-cm-diameter, 400 g sphere is released from rest at the top of a 2.1-m-long, 25° incline. It rolls, without slipping, to the bottom.
   a. What is the sphere’s angular velocity at the bottom of the incline?
   b. What fraction of its kinetic energy is rotational?

Section 12.10 The Vector Description of Rotational Motion

38. Evaluate the cross products \( \vec{A} \times \vec{B} \) and \( \vec{C} \times \vec{D} \).

![Figure EX12.38](image)

39. Evaluate the cross products \( \vec{A} \times \vec{B} \) and \( \vec{C} \times \vec{D} \).

![Figure EX12.39](image)

40. a. What is \((\vec{i} \times \vec{j}) \times \vec{i}\)?
   b. What is \(i \times (j \times i)\)?

41. a. What is \(\vec{i} \times (\vec{j} \times \vec{k})\)?
   b. What is \((\vec{i} \times \vec{j}) \times \vec{k}\)?

42. \[ \vec{A} = 3\vec{i} + \vec{j} \text{ and vector } \vec{B} = 3\vec{i} - 2\vec{j} + 2\vec{k}.
   a. What is the cross product \(\vec{A} \times \vec{B}\)?
   b. Show vectors \(\vec{A}, \vec{B}, \text{ and } \vec{A} \times \vec{B}\) on a three-dimensional coordinate system.

43. Consider the vector \(\vec{C} = 3\vec{i}\).
   a. What is a vector \(\vec{D}\) such that \(\vec{C} \times \vec{D} = \vec{0}\)?
   b. What is a vector \(\vec{E}\) such that \(\vec{C} \times \vec{E} = 6\vec{k}\)?
   c. What is a vector \(\vec{F}\) such that \(\vec{C} \times \vec{F} = -3\vec{j}\)?

44. Force \(\vec{F} = -10\vec{j}\) N is exerted on a particle at \(\vec{r} = (5\vec{i} + 5\vec{j})\) m. What is the torque on the particle about the origin?

45. Force \(\vec{F} = (-10\vec{i} + 10\vec{j})\) N is exerted on a particle at \(\vec{r} = 5\vec{j}\) m. What is the torque on the particle about the origin?

46. What are the magnitude and direction of the angular momentum relative to the origin of the 200 g particle in FIGURE EX12.46?

![Figure EX12.46](image)

Section 12.11 Angular Momentum of a Rigid Body

47. What are the magnitude and direction of the angular momentum relative to the origin of the 100 g particle in FIGURE EX12.47?

![Figure EX12.47](image)

48. What is the angular momentum of the 500 g rotating bar in FIGURE EX12.48?

![Figure EX12.48](image)

49. What is the angular momentum of the 2.0 kg, 4.0-cm-diameter rotating disk in FIGURE EX12.49?

50. How fast, in rpm, would a 100 g, 50-cm-diameter beach ball have to spin to have an angular momentum of 0.10 kg m²/s?

Problems

51. A 60-cm-diameter wheel is rolling along at 20 m/s. What is the speed of a point at the front edge of the wheel?

52. An equilateral triangle 5.0 cm on a side rotates about its center of mass at 120 rpm. What is the speed of one tip of the triangle?
67. A 3.0 kg block is attached to a string that is wrapped around a 2.0 kg, 4.0-cm-diameter hollow cylinder that is free to rotate. (Use Figure 12.34 but treat the cylinder as hollow.) The block is released 1.0 m above the ground.
   a. Use Newton's second law to find the speed of the block as it hits the ground.
   b. Use conservation of energy to find the speed of the block as it hits the ground.

68. A 60-cm-long, 500 g bar rotates in a horizontal plane on an axle that passes through the center of the bar. Compressed air is fed in through the axle, passes through a small hole down the length of the bar, and escapes as air jets from holes at the ends of the bar. The jets are perpendicular to the bar's axis. Starting from rest, the bar spins up to an angular velocity of 150 rpm at the end of 10 s.
   a. How much force does each jet of escaping air exert on the bar?
   b. If the axle is moved to one end of the bar while the air jets are unchanged, what will be the bar's angular velocity at the end of 10 seconds?

69. Flywheels are large, massive wheels used to store energy. They can be spun up slowly, then the wheel's energy can be released quickly to accomplish a task that demands high power. An industrial flywheel has a 1.5 m diameter and a mass of 250 kg. Its maximum angular velocity is 1200 rpm.
   a. A motor spins up the flywheel with a constant torque of 50 N·m. How long does it take the flywheel to reach top speed?
   b. How much energy is stored in the flywheel?
   c. The flywheel is disconnected from the motor and connected to a machine to which it will deliver energy. Half the energy stored in the flywheel is delivered in 2.0 s. What is the average power delivered to the machine?
   d. How much torque does the flywheel exert on the machine?

70. The two blocks in Figure P12.70 are connected by a massless rope that passes over a pulley. The pulley is 12 cm in diameter and has a mass of 2.0 kg. As the pulley turns, friction at the axle exerts a torque of magnitude 0.50 N·m. If the blocks are released from rest, how long does it take the 4.0 kg block to reach the floor?

71. Blocks of mass \( m_1 \) and \( m_2 \) are connected by a massless string that passes over the pulley in Figure P12.71. The pulley turns on frictionless bearings. Mass \( m_1 \) slides on a horizontal, frictionless surface. Mass \( m_2 \) is released while the blocks are at rest.
   a. Assume the pulley is massless. Find the acceleration of \( m_1 \) and the tension in the string. This is a Chapter 7 review problem.

72. Suppose the pulley has mass \( m_p \) and radius \( R \). Find the acceleration of \( m_1 \) and the tensions in the upper and lower portions of the string. Verify that your answers agree with part a if you set \( m_p = 0 \).

73. The 2.0 kg, 30-cm-diameter disk in Figure P12.72 is spinning at 300 rpm. How much friction force must the brake apply to the rim to bring the disk to a halt in 3.0 s?

74. The connecting tunnel in Example 12.11 has a mass of 50,000 kg.
   a. How far from the 100,000 kg rocket is the center of mass of the entire structure?
   b. What is the structure's angular velocity after 30 s?

75. A hollow sphere is rolling along a horizontal floor at 5.0 m/s when it comes to a 30° incline. How far up the incline does it roll before reversing direction?

76. Masses \( M \) and \( m \) are joined together by a massless, rigid rod of length \( L \). They rotate about a perpendicular axis at distance \( x \) from mass \( M \).
   a. For rotation at angular velocity \( \omega \), for what \( x \) does this rotating barbell have minimum rotational energy?
   b. What is the physical significance of this value of \( x \)?

77. A 5.0 kg, 60-cm-diameter disk rotates on an axle passing through one edge. The axle is parallel to the floor. The cylinder is held with the center of mass at the same height as the axle, then released.
   a. What is the cylinder's initial angular acceleration?
   b. What is the cylinder's angular velocity when it is directly below the axle?

78. A long, thin rod of mass \( M \) and length \( L \) is standing straight up on a table. Its lower end rotates on a frictionless pivot. A very slight push causes the rod to fall over. As it hits the table, what are (a) the angular velocity and (b) the speed of the tip of the rod?

79. A sphere of mass \( M \) and radius \( R \) is rigidly attached to a thin rod of radius \( r \) that passes through the sphere at distance \( \frac{1}{2} R \) from the center. A string wrapped around the rod pulls with tension \( T \). Find an expression for the sphere's angular acceleration. The rod's moment of inertia is negligible.

80. You've been given a pulley for your birthday. It's a fairly big pulley, 12 cm in diameter and with a mass of 2.0 kg. You get to
wondering whether the pulley is uniform. That is, is the mass evenly distributed, or is it concentrated toward the center or near the rim? To find out, you hang the pulley on a hook, wrap a string around it several times, and suspend your 1.0 kg physics book 1.0 m above the floor. With your stopwatch, you find that it takes 0.71 s for your book to hit the floor. What can you conclude about the pulley?

81. A satellite follows the elliptical orbit shown. The only force on the satellite is the gravitational attraction of the planet. The satellite's speed at point a is 8000 m/s.
   a. Is there any torque on the satellite? Explain.
   b. What is the satellite's speed at point b?
   c. What is the satellite's speed at point c?

82. FIGURE P12.82 shows two balls of clay approaching each other.
   a. Calculate the total angular momentum relative to the origin at this instant.
   b. Calculate the total angular momentum an instant before they collide.
   c. Calculate the total angular momentum an instant after the collision.

83. A 2.0 kg wood block hangs from the bottom of a 1.0 kg, 1.0-m-long rod. The block and rod form a pendulum that swings on a frictionless pivot at the top end of the rod. A 10 g bullet is fired into the block, where it sticks, causing the pendulum to swing out to a 30° angle. What was the speed of the bullet? You can treat the wood block as a particle.

84. A 10 g bullet traveling at 400 m/s strikes a 10 kg, 1.0-m-wide door at the edge opposite the hinge. The bullet embeds itself in the door, causing the door to swing open. What is the angular velocity of the door just after impact?

85. A solid sphere of radius \( R \) is placed at a height of 30 cm on a 15° slope. It is released and rolls, without slipping, to the bottom.
   a. From what height should a circular loop of radius \( R \) be released on the same slope in order to equal the sphere's speed at the bottom?
   b. Can a circular loop of different diameter be released from a height of 30 cm and match the sphere's speed at the bottom? If so, what is the diameter? If not, why not?

86. II A 2.0 kg, 20-cm-diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at opposite ends of a diagonal, and stick. What is the turntable's angular velocity, in rpm, just after this event?

87. II A 200 g, 40-cm-diameter turntable rotates on frictionless bearings at 60 rpm. A 20 g block sits at the center of the turntable. A compressed spring shoots the block radially outward along a frictionless groove in the surface of the turntable. What is the turntable's rotation angular velocity when the block reaches the outer edge?

88. II A merry-go-round is a common piece of playground equipment. A 3.0-m-diameter merry-go-round with a mass of 250 kg is spinning at 20 rpm. John runs tangent to the merry-go-round at 5.0 m/s, in the same direction that it is turning, and jumps onto the outer edge. John's mass is 30 kg. What is the merry-go-round's angular velocity, in rpm, after John jumps on?

89. II A 200 g toy car is placed on a narrow 60-cm-diameter track with wheel grooves that keep the car going in a circle. The 1.0 kg track is free to turn on a frictionless, vertical axis. The spokes have negligible mass. After the car's switch is turned on, it soon reaches a steady speed of 0.75 m/s relative to the track. What then is the track's angular velocity, in rpm?

90. II A 45 kg figure skater is spinning on the toes of her skates at 1.0 rev/s. Her arms are outstretched as far as they will go. In this orientation, the skater can be modeled as a cylindrical torso (40 kg, 20 cm average diameter, 160 cm tall) plus two rod-like arms (2.5 kg each, 66 cm long) attached to the outside of the torso. The skater then raises her arms straight above her head, where she appears to be a 45 kg, 20-cm-diameter, 200-cm-tall cylinder. What is her new rotation frequency, in revolutions per second?

**Challenge Problems**

91. The marble rolls down a track and around a loop-the-loop of radius \( R \). The marble has mass \( m \) and radius \( r \). What minimum height \( h \) must the track have for the marble to make it around the loop-the-loop without falling off?

92. FIGURE CP12.92 shows a triangular block of Swiss cheese sitting on a cheese board. You and your friends start to wonder what will happen if you slowly tilt the board, increasing angle \( \theta \). Emily thinks the cheese will start to slide before it topples over. Fred thinks it will topple before starting to slide. Some quick Internet research on your part reveals that the coefficient of static friction of Swiss cheese on wood is 0.90. Who is right?